Fully Probabilistic Design Promises and Prospects

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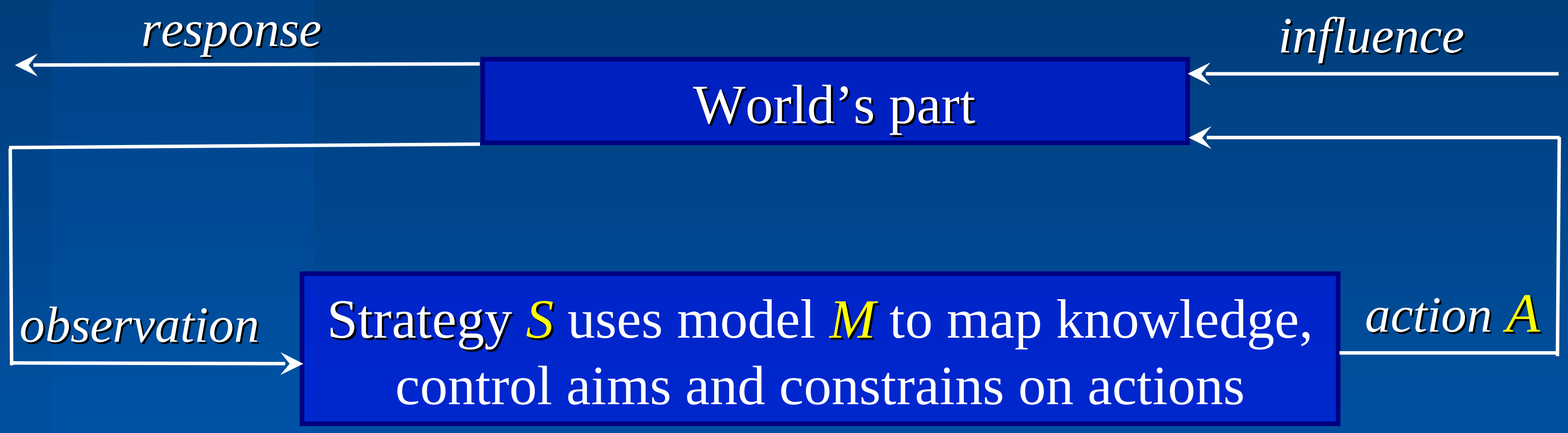
Control under uncertainty



Behavior B: all considered observations, actions, and influences, responses, model parameters

non-empty uncertainty

Closed-loop *probabilistic model* of behavior B is $C(B) = M(B) \times S(B)$

Optimal control design

Closed-loop model is considered: $C(B) = M(B) \times S(B)$

A loss $L(B,C(B))$ is chosen that expresses desirability of behavior B with respect to control aims.

The *optimal control strategy* ${}^{\text{qt}}S_L$ for the given L is then defined :

$$ {}^{\text{qt}}S_L(B) \in \text{Arg}\min_S \int L(B,C(B))\, C(B)\, \mathrm{d}B $$

The strategy ${}^{\text{qt}}S_L(B)$ and the model $M(B)$ provide a description of the *'ideal' closed control loop* ${}^{I}C_L\,(B)$:

$$ {}^{I}C_L\,(B) \equiv M\,(B) \times {}^{\text{qt}}S_L\,(B) \quad \text{for the given loss } L(B,C(B)) $$

FPD and its justification

- Consider a set of losses L^* leading to the same ideal ${}^{I}C(B)$, $\forall\, L \in L^*$.
- Find a representative L of L^* whose values $L(B, {}^{I}C(B))$ do not depend on the realization of behavior B.

Theorem: The function $L = \ln\,(C\,/\,{}^{I}C)$ is such a representative.

Hence, Kullback-Leibler divergence $D(C\,||\,{}^{I}C)$ of C on ${}^{I}C$ represents control designs leading to the given ideal closed control loop ${}^{I}C$ *i.e.*, ${}^{\text{qt}}S(B) \in \text{Arg}\min_S D(C\,||\,{}^{I}C)$.

This control design is called *Fully Probabilistic Design* (*FPD*).

Properties and promises of FPD

- Minimization in FPD has an explicit solution :
  - comparing to dynamic programming, an approximation of the underlined equations is *simpler*. An approximation may rely on MC integration & interpolation of multivariate function.
- Optimal strategy is randomized :
  - a dither noise supporting exploration is ‘naturally’ added.
- Support of the optimal strategy is a part of its ideal counterpart :
  - unlike loss function, ideal ${}^{I}C$ describes both control aims and constrains.

Properties and promises of FPD (cont.)

- An *aim-expressing* 'ideal' ${}^{I}C(B)$ is nothing but *distribution*. Hence, all operations supported for models can be applied to ${}^{I}C(B)$ as well.

  The most significant are:

  - approximation of unfeasible distribution by a feasible one, thus aims can be approximately quantified by a feasible ideal
  - extension of a partial knowledge into distribution, thus partially described aims can be completed in a meaningful way
  - merging of several distribution into a single representative, thus multiple, incompatible aims can be systematically merged

Properties and promises of FPD (end)

- To any control task, we can formulate FPD so that its solution will be *arbitrarily close* to the standard control design solution.

Proof: Let us perform a standard control design on a set of strategies S^* yielding a *constrained expected entropy*. Then,

$$\text{qt } S_L(B) \in \text{Argmin}_S \, T \int [L(B)/T + \ln(C(B))] \, C(B) \, \mathrm{d}B$$

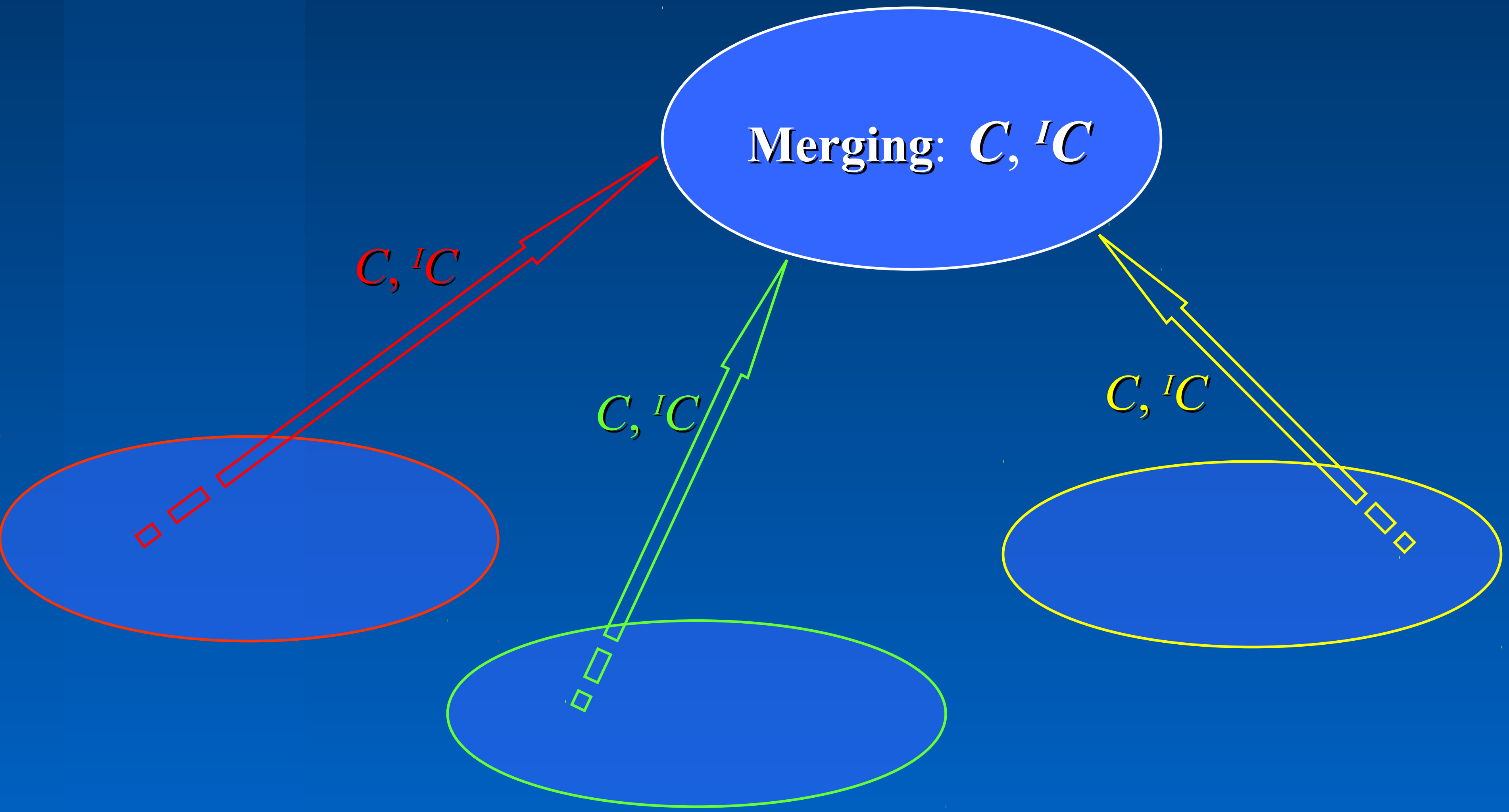
$$= \text{Argmin}_S \, D(C \,||\, {}^{I}C_L) \text{ , where}$$

$${}^{I}C_L \propto M \exp(- L/T),$$

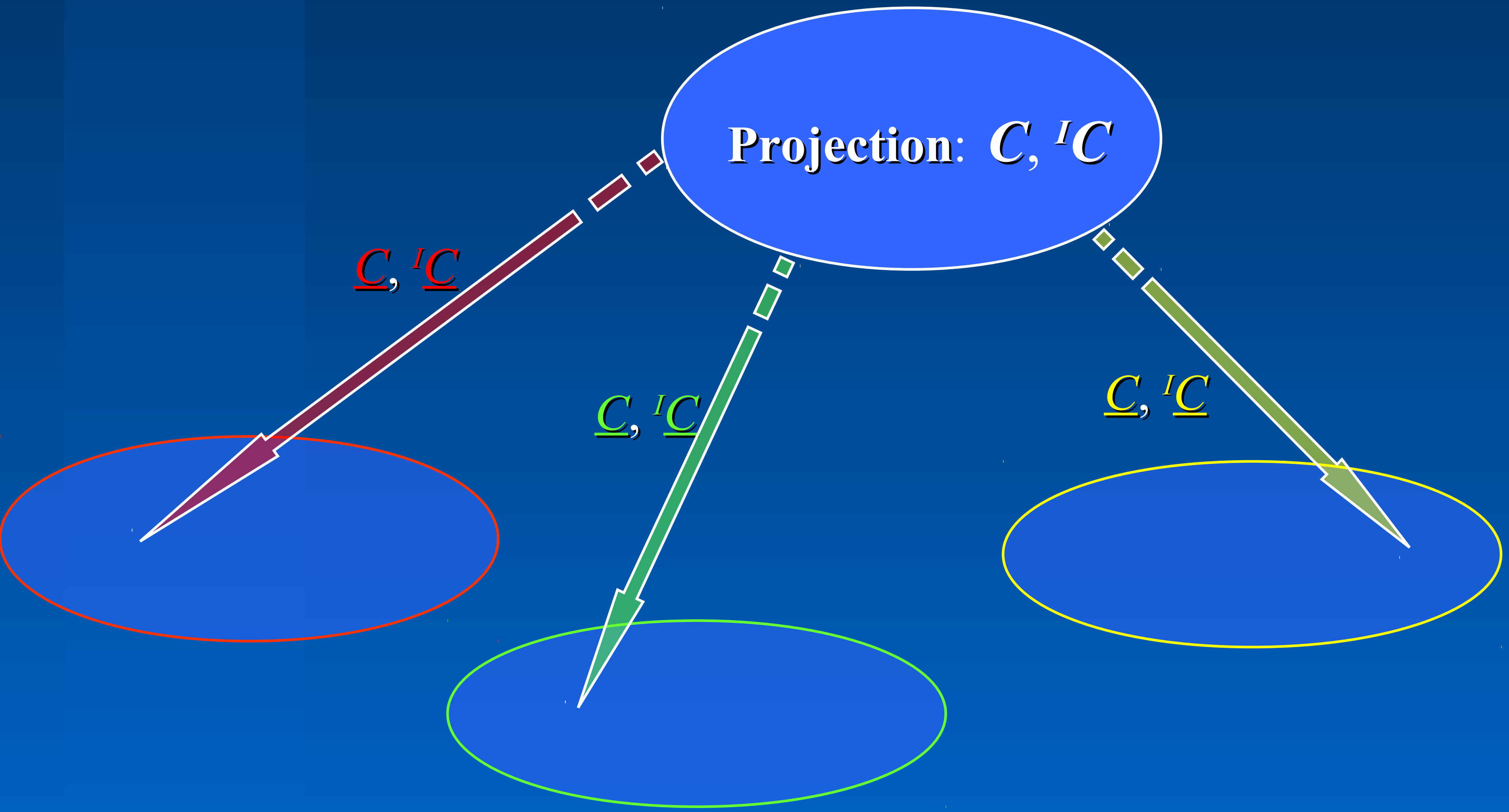
$T \geq 0$ is Kuhn-Tucker multiplier, or temperature in Boltzmann machine.

- ➢ unrealistic demands can be voided, for instance, quadratic loss L is nonsense for heavy tailed models M
- ➢ LQ design is re-interpreted in terms of normal ideal ${}^{I}C$. This provides better adaptive controllers via loss-weights adaptation

Prospect of FPD: flatly-distributed control



Prospect of FPD: flatly-distributed control



Conclusions (prospects cont.)

A complete framework is set, but a lot remains to be done, in particular :

- to *refine* axiomatic basis
- to *investigate* when FPD leads to linear problem (like in KL control)
- to *develop* all algorithmic steps for a rich class of models and ideals
- to *adapt* the art of approximate dynamic programming to FPD
- to *elaborate* use of learned model to construction of ideal
- to *exploit* potential of flat cooperation
- to *go* beyond LQ and mixtures of LQ's designs …

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