

Lecture 1: Introduction to Machine Learning

4F13: Machine Learning

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What is machine learning?

- *Machine learning is an interdisciplinary field focusing on both the mathematical foundations and practical applications of systems that learn, reason and act.*
- Other related terms: Pattern Recognition, Neural Networks, Data Mining, Statistical Modelling ...
- Using ideas from: Statistics, Computer Science, Engineering, Applied Mathematics, Cognitive Science, Psychology, Computational Neuroscience, Economics
- **The goal of these lectures:** to introduce important concepts, models and algorithms in machine learning.
- **For more:** We have organised a “Tutorial Lecture Series on Machine Learning” with a series of guest lecturers (Thursdays, 4-6pm in LT2). Go to talks.cam.ac.uk, search for “Machine Learning” for various local reading groups, lectures, and seminars.

Warning!

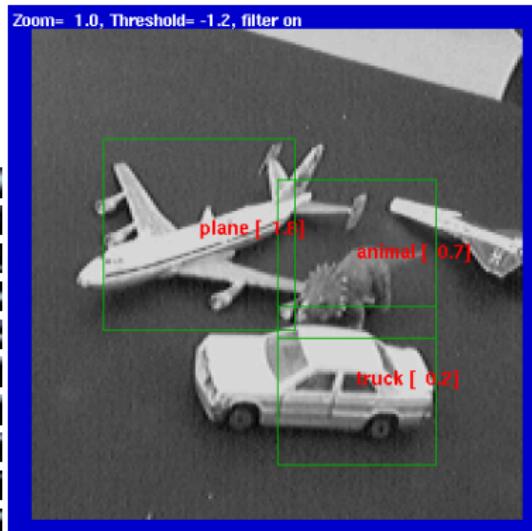
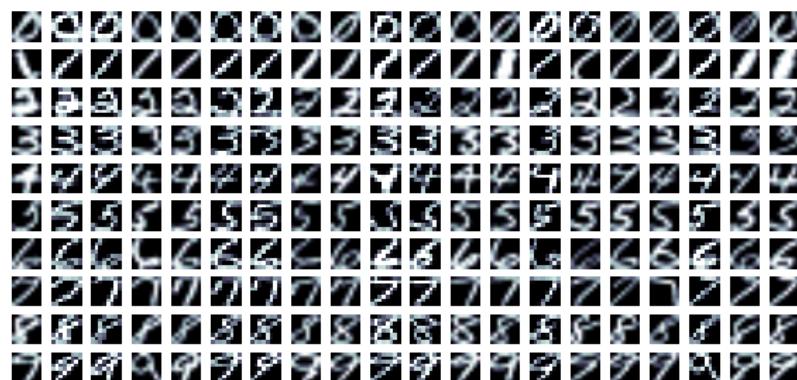
Lecture 1 will overlap somewhat with my lectures in 3f3: Pattern Processing—but don't despair, a lot of new material later!

What is machine learning useful for?

Automatic speech recognition



Computer vision: e.g. object, face and handwriting recognition



(NORB image from Yann LeCun)

Information retrieval

Google Search: Unsupervised Learning [http://www.google.com/search?q=Unsupervised+Learning&sourceid=fire...](http://www.google.com/search?q=Unsupervised+Learning&sourceid=firefox&rlz=1C1GCEU_enUS380US380)

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NIPS'98 Workshop - Integrating Supervised and Unsupervised Learning, Friday, December 4, 1998 ... 4:45-5:30, Theories of Unsupervised Learning and Missing Values. ...
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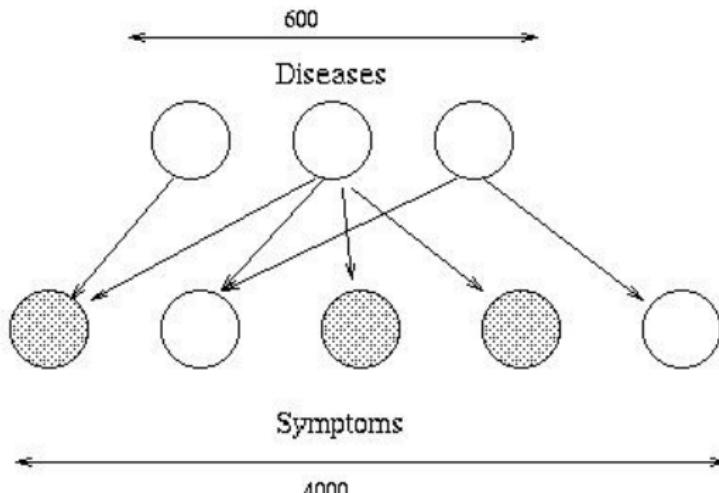
Web Pages

Retrieval
Categorisation
Clustering
Relations between pages

Financial prediction

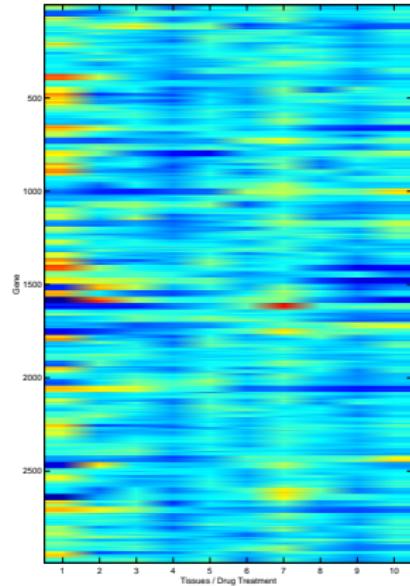
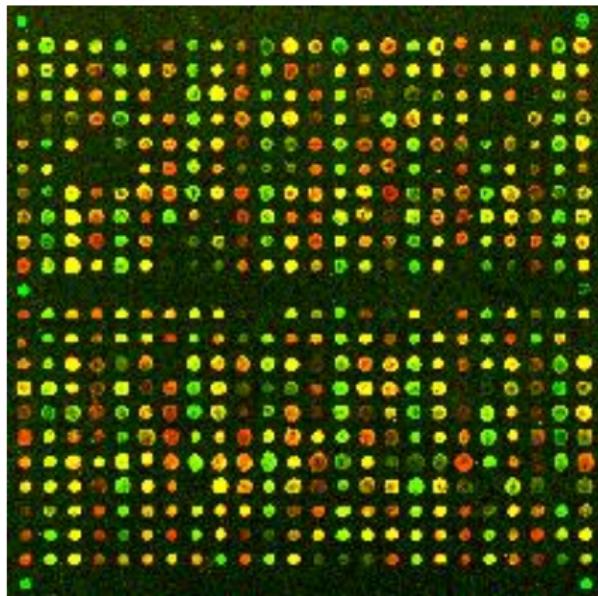


Medical diagnosis



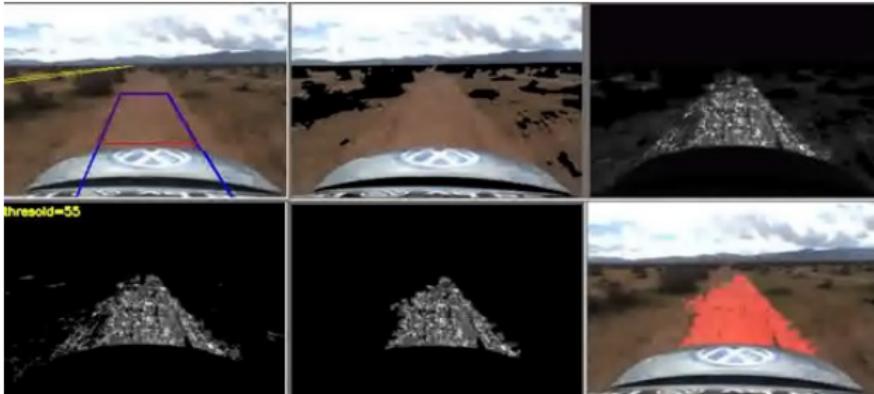
(image from Kevin Murphy)

Bioinformatics



e.g. modelling gene microarray data, protein structure prediction

Robotics



DARPA \$2m Grand Challenge

Ghahramani & Rasmussen (CUED)

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Movie recommendation systems



Challenge: to improve the accuracy of movie preference predictions

Netflix \$1m Prize. Competition started Oct 2, 2006 and still ongoing.

Three Types of Learning

Imagine an organism or machine which experiences a series of sensory inputs:

$$x_1, x_2, x_3, x_4, \dots$$

Supervised learning: The machine is also given **desired outputs** y_1, y_2, \dots , and its goal is to learn to **produce the correct output** given a new input.

Unsupervised learning: The goal of the machine is to **build a model** of x that can be used for reasoning, decision making, predicting things, communicating etc.

Reinforcement learning: The machine can also produce **actions** a_1, a_2, \dots which affect the state of the world, and receives **rewards (or punishments)** r_1, r_2, \dots . Its goal is to learn to act in a way that **maximises rewards** in the long term.

(In this course we'll focus mostly on unsupervised learning and reinforcement learning.)

Key Ingredients

Data

We will represent data by vectors in some vector space¹

Let \mathbf{x} denote a **data point** with elements $\mathbf{x} = (x_1, x_2, \dots, x_D)$

The elements of \mathbf{x} , e.g. x_d , represent measured (observed) **features** of the data point; D denotes the number of measured features of each point.

The **data set** \mathcal{D} consists of N data points:

$$\mathcal{D} = \{\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \dots, \mathbf{x}^{(N)}\}$$

¹This assumption can be relaxed.

Key Ingredients

Data

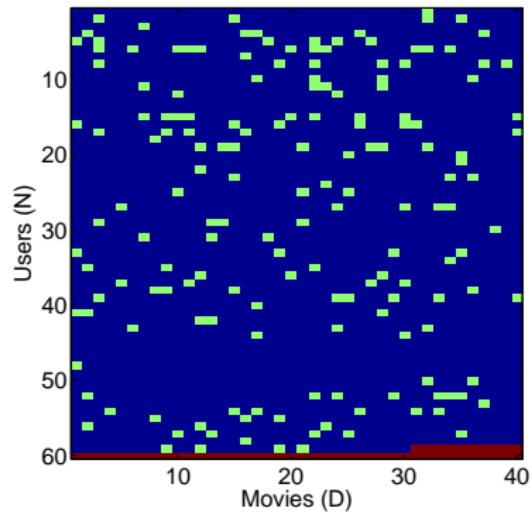
Let $\mathbf{x} = (x_1, x_2, \dots, x_D)$ denote a **data point**, and $\mathcal{D} = \{\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(N)}\}$, a **data set**

Predictions

We are generally interested in predicting something based on the observed data.

Given \mathcal{D} what can we say about $\mathbf{x}^{(N+1)}$?

Given \mathcal{D} and $x_1^{(N+1)}, x_2^{(N+1)}, \dots, x_{D-1}^{(N+1)}$, what can we say about $x_D^{(N+1)}$?



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Model

To make predictions, we need to make some **assumptions**. We can often express these assumptions in the form of a **model**, with some **parameters**, θ .

Given data \mathcal{D} , we learn the model parameters θ , from which we can predict new data points.

The model can often be expressed as a **probability distribution over data points**

Basic Rules of Probability

Let X be a random variable taking values x in some set \mathcal{X} .

Probabilities are non-negative $P(X = x) \geq 0 \forall x$.

Probabilities normalise: $\sum_{x \in \mathcal{X}} P(X = x) = 1$ for distributions if x is a discrete variable and $\int_{-\infty}^{+\infty} p(x)dx = 1$ for probability densities over continuous variables

The **joint probability** of $X = x$ and $Y = y$ is: $P(X = x, Y = y)$.

The **marginal probability** of $X = x$ is: $P(X = x) = \sum_y P(X = x, y)$, assuming y is discrete. I will generally write $P(x)$ to mean $P(X = x)$.

The **conditional probability** of x given y is: $P(x|y) = P(x, y)/P(y)$

Bayes Rule:

$$P(x, y) = P(x)P(y|x) = P(y)P(x|y)$$

\Rightarrow

$$P(y|x) = \frac{P(x|y)P(y)}{P(x)}$$

Warning: I will not be obsessively careful in my use of p and P for probability density and probability distribution. Should be obvious from context.

Information, Probability and Entropy

Information is the **reduction of uncertainty**. How do we measure uncertainty?

Some axioms (informally):

- if something is certain, its uncertainty = 0
- uncertainty should be maximum if all choices are equally probable
- uncertainty (information) should add for independent sources

This leads to a discrete random variable X having uncertainty equal to the **entropy** function:

$$H(X) = - \sum_{x \in \mathcal{X}} P(X = x) \log P(X = x)$$

measured in *bits* (binary digits) if the base 2 logarithm is used or *nats* (natural digits) if the natural (base e) logarithm is used.

Some Definitions Relating to Information Theory

- **Surprise** (for event $X = x$): $-\log P(X = x)$
- **Entropy** = average surprise: $H(X) = -\sum_{x \in \mathcal{X}} P(X = x) \log P(X = x)$
- **Conditional entropy**

$$H(X|Y) = -\sum_x \sum_y P(x, y) \log P(x|y)$$

- **Mutual information**

$$I(X; Y) = H(X) - H(X|Y) = H(Y) - H(Y|X) = H(X) + H(Y) - H(X, Y)$$

- Independent random variables: $P(x, y) = P(x)P(y) \forall x \forall y$

How do we relate information theory and probabilistic modelling?

The source coding problem

Imagine we have a set of symbols $\mathcal{X} = \{a, b, c, d, e, f, g, h\}$.

We want to transmit these symbols over some binary communication channel, i.e. using a sequence of **bits** to represent the symbols.

Since we have 8 symbols, we could use 3 bits per symbol ($2^3 = 8$). For example:
 $a = 000, b = 001, c = 010, \dots, h = 111$

Is this optimal?

What if some symbol, a , is much more probable than other symbols, e.g. f ?
Shouldn't we use fewer bits to transmit the more probable symbols?

Think of a discrete variable X taking on values in \mathcal{X} , having probability distribution $P(X)$.

How does the probability distribution $P(X)$ relate to the number of bits we need for each symbol to optimally and losslessly transmit symbols from \mathcal{X} ?

Shannon's Source Coding Theorem

A discrete random variable X , distributed according to $P(X)$ has **entropy**:

$$H(X) = - \sum_{x \in \mathcal{X}} P(x) \log_2 P(x)$$

Shannon's source coding theorem: Consider a random variable X , with entropy $H(X)$. A sequence of n independent draws from X can be losslessly compressed into a minimum expected code of length $n\mathcal{L}$ bits, where $H(X) \leq \mathcal{L} < H(X) + \frac{1}{n}$.

If each symbol is given a code length $l(x) = -\log_2 Q(x)$ then the expected per-symbol length \mathcal{L}_Q of the code is

$$\mathcal{L}_Q = \sum_x P(x)l(x) = - \sum_x P(x) \log_2 Q(x) = H(X) + KL(P\|Q),$$

where the **relative-entropy** or **Kullback-Leibler divergence** is

$$KL(P\|Q) = \sum_x P(x) \log_2 \frac{P(x)}{Q(x)} \geq 0$$

Take home message: better probabilistic models \equiv more efficient codes

Some distributions

Univariate Gaussian density ($x \in \mathbb{R}$):

$$p(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

Multivariate Gaussian density ($\mathbf{x} \in \mathbb{R}^D$):

$$p(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\Sigma}) = |2\pi\boldsymbol{\Sigma}|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right\}$$

Bernoulli distribution ($x \in \{0, 1\}$):

$$p(x|\theta) = \theta^x(1-\theta)^{1-x}$$

Discrete distribution ($x \in \{1, \dots, L\}$):

$$p(x|\theta) = \prod_{\ell=1}^L \theta_\ell^{\delta(x, \ell)}$$

where $\delta(a, b) = 1$ iff $a = b$, and $\sum_{\ell=1}^L \theta_\ell = 1$ and $\theta_\ell \geq 0 \ \forall \ell$.

Some distributions (cont)

Uniform ($x \in [a, b]$):

$$p(x|a, b) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

Gamma ($x \geq 0$):

$$p(x|a, b) = \frac{b^a}{\Gamma(a)} x^{a-1} \exp\{-bx\}$$

Beta ($x \in [0, 1]$):

$$p(x|\alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$$

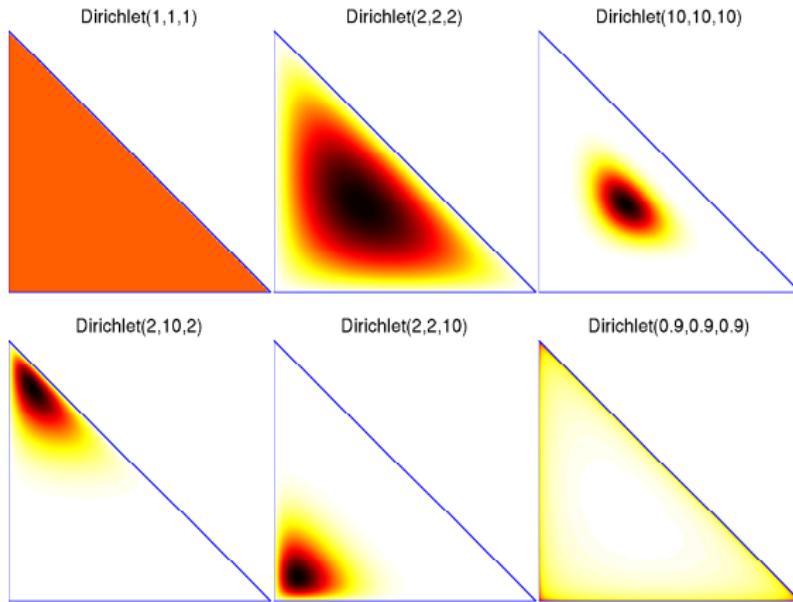
where $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$ is the gamma function, a generalisation of the factorial: $\Gamma(n) = (n-1)!$.

Dirichlet ($\mathbf{p} \in \mathbb{R}^D, p_d \geq 0, \sum_{d=1}^D p_d = 1$):

$$p(\mathbf{p}|\boldsymbol{\alpha}) = \frac{\Gamma(\sum_{d=1}^D \alpha_d)}{\prod_{d=1}^D \Gamma(\alpha_d)} \prod_{d=1}^D p_d^{\alpha_d-1}$$

Dirichlet Distributions

Examples of Dirichlet distributions over $\mathbf{p} = (p_1, p_2, p_3)$ which can be plotted in 2D since $p_3 = 1 - p_1 - p_2$:



Other distributions you should know about...

Exponential family of distributions:

$$P(\mathbf{x}|\boldsymbol{\theta}) = f(\mathbf{x}) g(\boldsymbol{\theta}) \exp \left\{ \boldsymbol{\Phi}(\boldsymbol{\theta})^\top \mathbf{u}(\mathbf{x}) \right\}$$

where $\boldsymbol{\Phi}(\boldsymbol{\theta})$ is the vector of *natural parameters*, \mathbf{u} are *sufficient statistics*

- Binomial
- Multinomial
- Poisson
- Student t distribution
- ...

End Notes

It is very important that you *understand* all the material in the following cribsheet:

<http://learning.eng.cam.ac.uk/zoubin/ml06/cribsheet.pdf>

Here is a useful statistics / pattern recognition glossary:

<http://research.microsoft.com/~minka/statlearn/glossary/>