3F3: Signal and Pattern Processing

Lecture 1: Introduction

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Pattern Processing

Other related terms:

- Pattern Recognition
- Neural Networks
- Data Mining
- Statistical Modelling
- Machine Learning

Learning:
The view from different fields

- Engineering: signal processing, system identification, adaptive and optimal control, information theory, robotics, ...
- Computer Science: Artificial Intelligence, computer vision, information retrieval, ...
- Statistics: learning theory, data mining, learning and inference from data, ...
- Cognitive Science and Psychology: perception, movement control, reinforcement learning, mathematical psychology, computational linguistics, ...
- Computational Neuroscience: neuronal networks, neural information processing, ...
- Economics: decision theory, game theory, operational research, ...

Different fields, Convergent ideas

- The same set of ideas and mathematical tools have emerged in many of these fields, albeit with different emphases.

- *Machine learning is an interdisciplinary field focusing on both the mathematical foundations and practical applications of systems that learn, reason and act.*

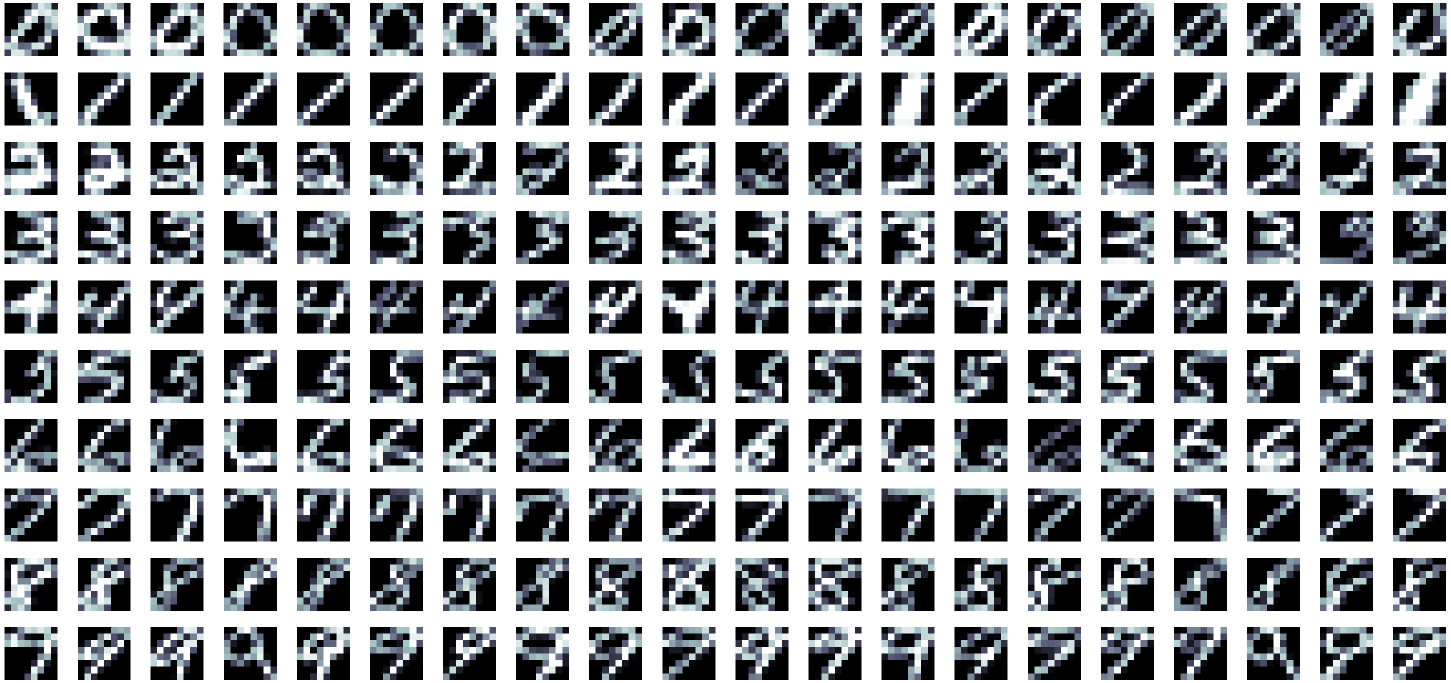
- The goal of these lectures: to introduce very basic concepts, models and algorithms.

- For more on this topic: I will be teaching a 4th year module called "4F13: Machine Learning" in Michaelmas 2007. I also offer 4th year projects on this topic.

Applications of Pattern Processing and Machine Learning

Automatic speech recognition

Computer vision: object, face and handwriting recognition



(NORB image from Yann LeCun)

Information retrieval

Google Search: Unsupervised Learning http://www.google.com/search?q=Unsupervised+Learning&sourceid=fir...

Google

Web Images Groups News Froogle **more »**

Unsupervised Learning Search Advanced Search Preferences

Web Results **1** - **10** of about **150,000** for **Unsupervised Learning**. (**0.27** seconds)

Mixture modelling, Clustering, Intrinsic classification ...
Mixture Modelling page. Welcome to David Dowe's clustering, mixture modelling and **unsupervised learning** page. Mixture modelling (or **...**
www.csse.monash.edu.au/~dld/mixture.modelling.page.html - 26k - 4 Oct 2004 - Cached - Similar pages

ACL'99 Workshop -- **Unsupervised Learning** in Natural Language **...**
PROGRAM. ACL'99 Workshop **Unsupervised Learning** in Natural Language Processing. University of Maryland June 21, 1999. Endorsed by SIGNLL **...**
www.ai.sri.com/~kehler/unsup-acl-99.html - 5k - Cached - Similar pages

Unsupervised learning and Clustering
cgm.cs.mcgill.ca/~soss/cs644/projects/wijhe/ - 1k - Cached - Similar pages

NIPS*98 Workshop - Integrating Supervised and **Unsupervised ...**
NIPS*98 Workshop "Integrating Supervised and **Unsupervised Learning**" Friday, December 4, 1998. **...** 4:45-5:30, Theories of **Unsupervised Learning** and Missing Values. **...**
www-2.cs.cmu.edu/~mccallum/supunsup/ - 7k - Cached - Similar pages

NIPS Tutorial 1999
Probabilistic Models for **Unsupervised Learning** Tutorial presented at the 1999 NIPS Conference by Zoubin Ghahramani and Sam Roweis. **...**
www.gatsby.ucl.ac.uk/~zoubin/NIPStutorial.html - 4k - Cached - Similar pages

Gatsby Course: **Unsupervised Learning** : Homepage
Unsupervised Learning (Fall 2000). **...** Syllabus (resources page): 10/10 1 - Introduction to **Unsupervised Learning** Geoff project: (ps, pdf). **...**
www.gatsby.ucl.ac.uk/~quaid/course/ - 15k - Cached - Similar pages
[More results from www.gatsby.ucl.ac.uk]

[PDF] Unsupervised Learning of the Morphology of a Natural Language
File Format: PDF/Adobe Acrobat - View as HTML
Page 1. Page 2. Page 3. Page 4. Page 5. Page 6. Page 7. Page 8. Page 9. Page 10. Page 11. Page 12. Page 13. Page 14. Page 15. Page 16. Page 17. Page 18. Page 19 **...**
acl.ldc.upenn.edu/J/J01/J01-2001.pdf - Similar pages

Unsupervised Learning - The MIT Press
... From Bradford Books: **Unsupervised Learning** Foundations of Neural Computation Edited by Geoffrey Hinton and Terrence J. Sejnowski Since its founding in 1989 by **...**
mitpress.mit.edu/book-home.tcl?isbn=026258168X - 13k - Cached - Similar pages

[PS] Unsupervised Learning of Disambiguation Rules for Part of
File Format: Adobe PostScript - View as Text
Unsupervised Learning of Disambiguation Rules for Part of. Speech Tagging. Eric Brill. 1. **...** It is possible to use **unsupervised learning** to train stochastic. **...**
www.cs.jhu.edu/~brill/acl-wkshp.ps - Similar pages

The **Unsupervised Learning** Group (ULG) at UT Austin
The **Unsupervised Learning** Group (ULG). What ? The **Unsupervised Learning** Group (ULG) is a group of graduate students from the Computer **...**
www.lans.ece.utexas.edu/ulg/ - 14k - Cached - Similar pages

Gooooooooooogle ▶
Result Page: 1 2 3 4 5 6 7 8 9 10 **Next**

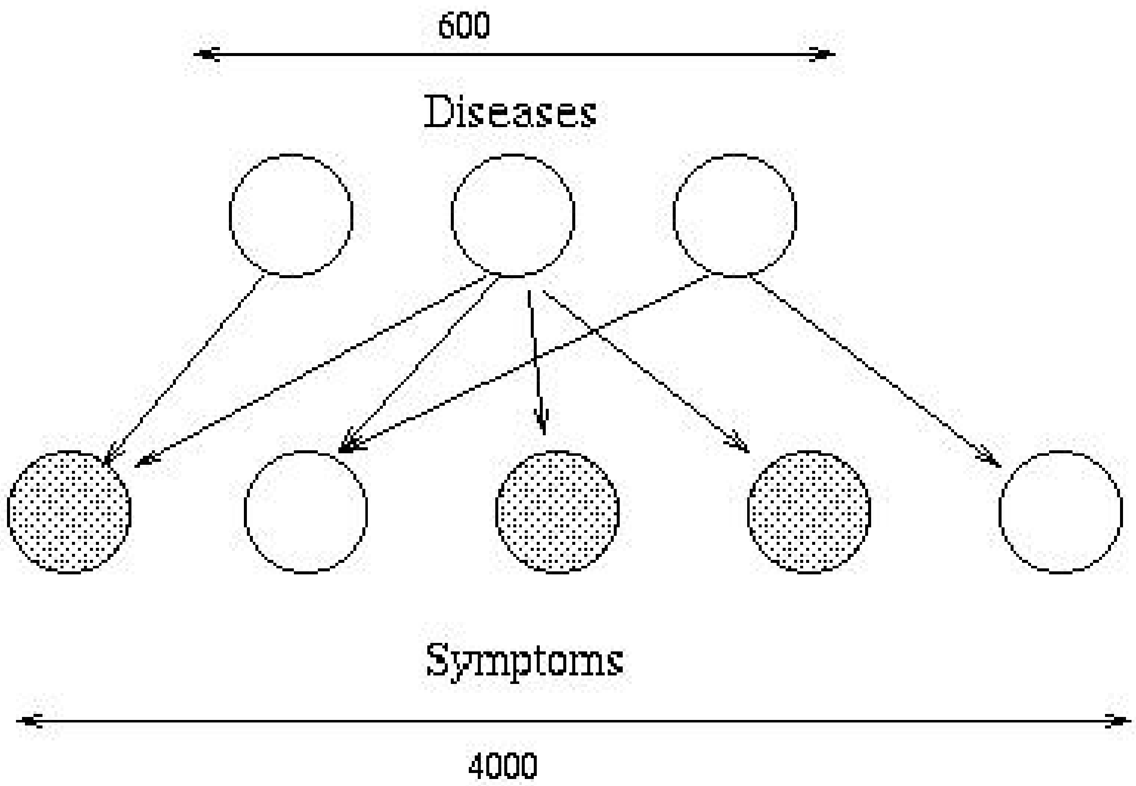
1 of 2 06/10/04 15:44

Web Pages

Retrieval
Categorisation
Clustering
Relations between pages

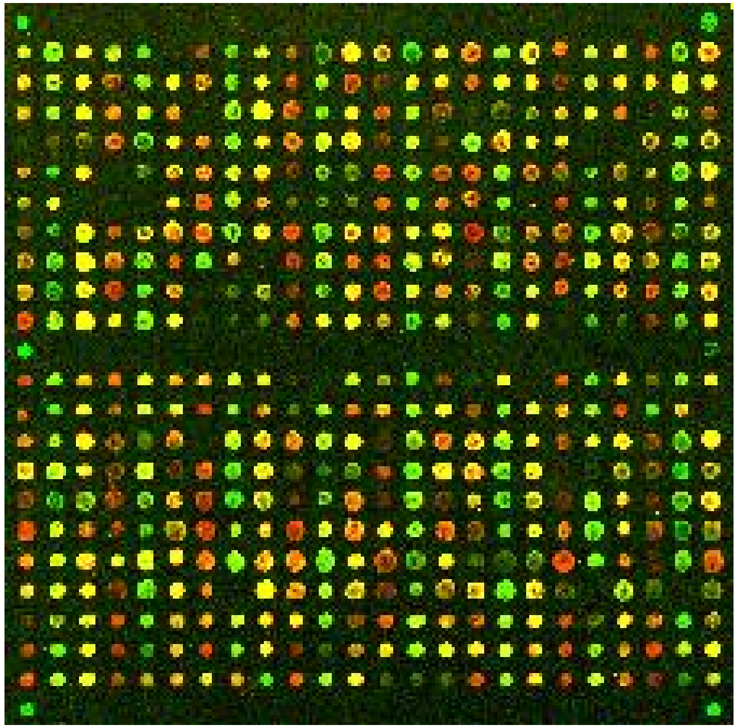
Financial Prediction

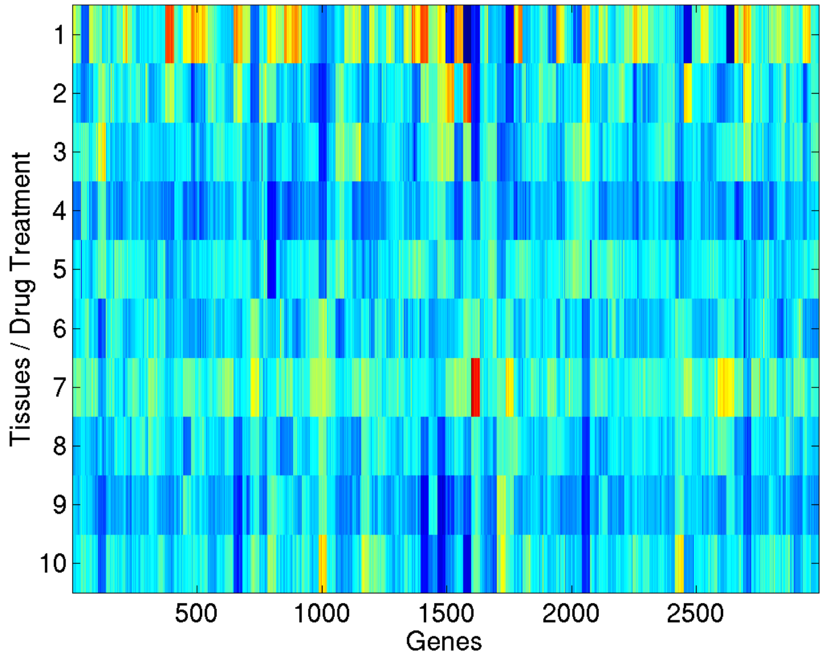
Medical diagnosis



(image from Kevin Murphy)

Gene microarrays and bioinformatics





Three Types of Learning

Imagine an organism or machine which experiences a series of sensory inputs:

$$x_1, x_2, x_3, x_4, \ldots$$

Supervised learning: The machine is also given desired outputs $y_1, y_2, \ldots$, and its goal is to learn to produce the correct output given a new input.

Unsupervised learning: The goal of the machine is to build a model of x that can be used for reasoning, decision making, predicting things, communicating etc.

Reinforcement learning: The machine can also produce actions $a_1, a_2, \ldots$ which affect the state of the world, and receives rewards (or punishments) $r_1, r_2, \ldots$. Its goal is to learn to act in a way that maximises rewards in the long term.

Three Problems

Over the next three lectures we will cover these three topics:

- Classification

- Regression

- Clustering

We will make extensive use of probability, statistics, calculus and linear algebra.

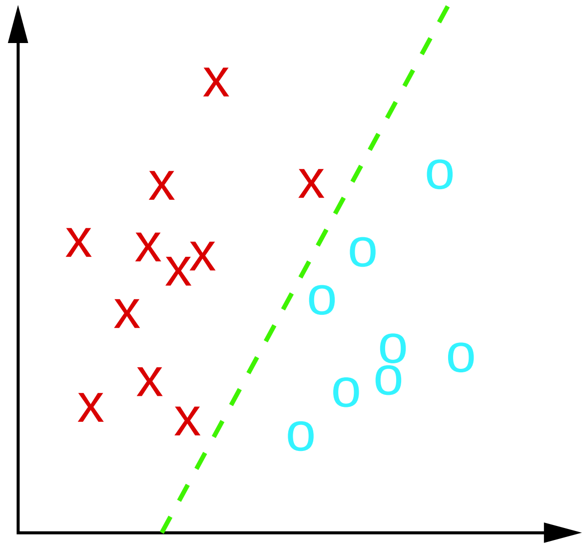
Classification

We will represent data by vectors in some vector space.
Let $\mathbf{x}$ denote a data point with elements $\mathbf{x} = (x_1, x_2, \ldots, x_D)$
The elements of $\mathbf{x}$, e.g. x_d, represent measured (observed) features of the data point; D denotes the number of measured features of each point.

The data set $\mathcal{D}$ consists of N pairs of data points and corresponding discrete class labels:

$$\mathcal{D} = \{(\mathbf{x}^{(1)}, y^{(1)}) \ldots, (\mathbf{x}^{(N)}, y^{(N)})\}$$

where $y^{(n)} \in \{1, \ldots, C\}$ and C is the number of classes.
The goal is to classify new inputs correctly (i.e. to *generalize*).



Examples:

- spam vs non-spam
- normal vs disease
- 0 vs 1 vs 2 vs 3 ... vs 9

Classification: Example Iris Dataset

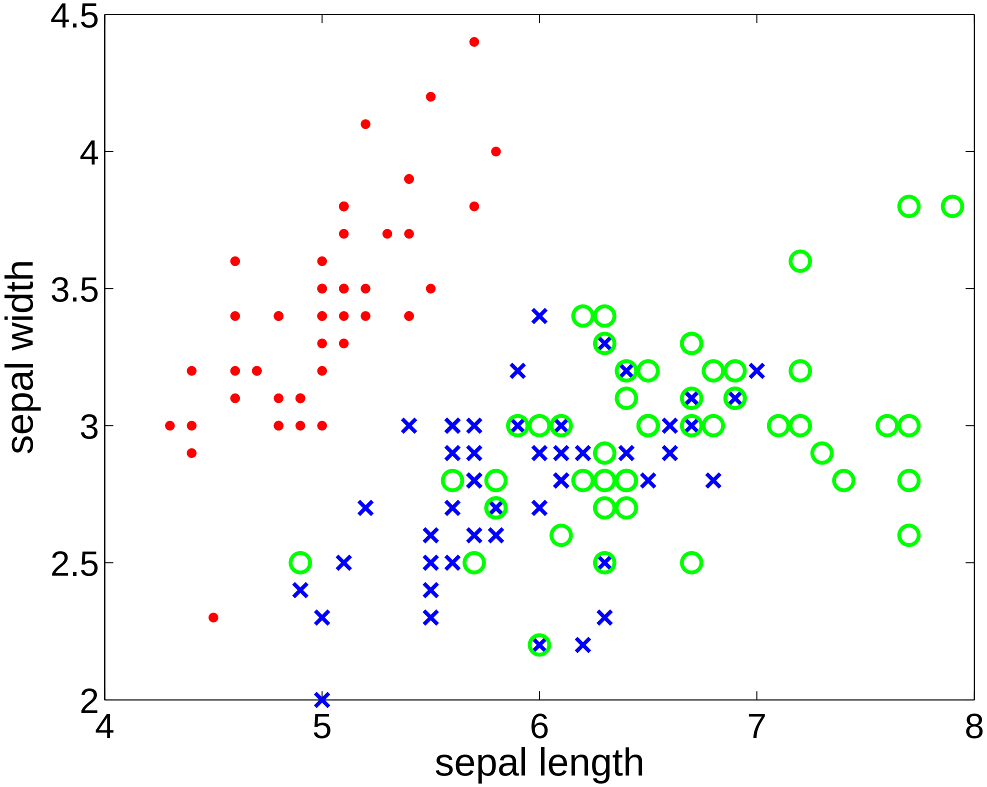
3 classes, 4 numeric attributes, 150 instances

A data set with 150 points and 3 classes. Each point is a random sample of measurements of flowers from one of three iris species—setosa, versicolor, and virginica—collected by Anderson (1935). Used by Fisher (1936) for linear discriminant function technique.

The measurements are sepal length, sepal width, petal length, and petal width in cm.

Data:

```
5.1,3.5,1.4,0.2,Iris-setosa

4.9,3.0,1.4,0.2,Iris-setosa

4.7,3.2,1.3,0.2,Iris-setosa

...

7.0,3.2,4.7,1.4,Iris-versicolor

6.4,3.2,4.5,1.5,Iris-versicolor

6.9,3.1,4.9,1.5,Iris-versicolor

...

6.3,3.3,6.0,2.5,Iris-virginica

5.8,2.7,5.1,1.9,Iris-virginica

7.1,3.0,5.9,2.1,Iris-virginica
```



Regression

Let $\mathbf{x}$ denote an input point with elements $\mathbf{x} = (x_1, x_2, \ldots, x_D)$. The elements of $\mathbf{x}$, e.g. x_d, represent measured (observed) features of the data point; D denotes the number of measured features of each point.

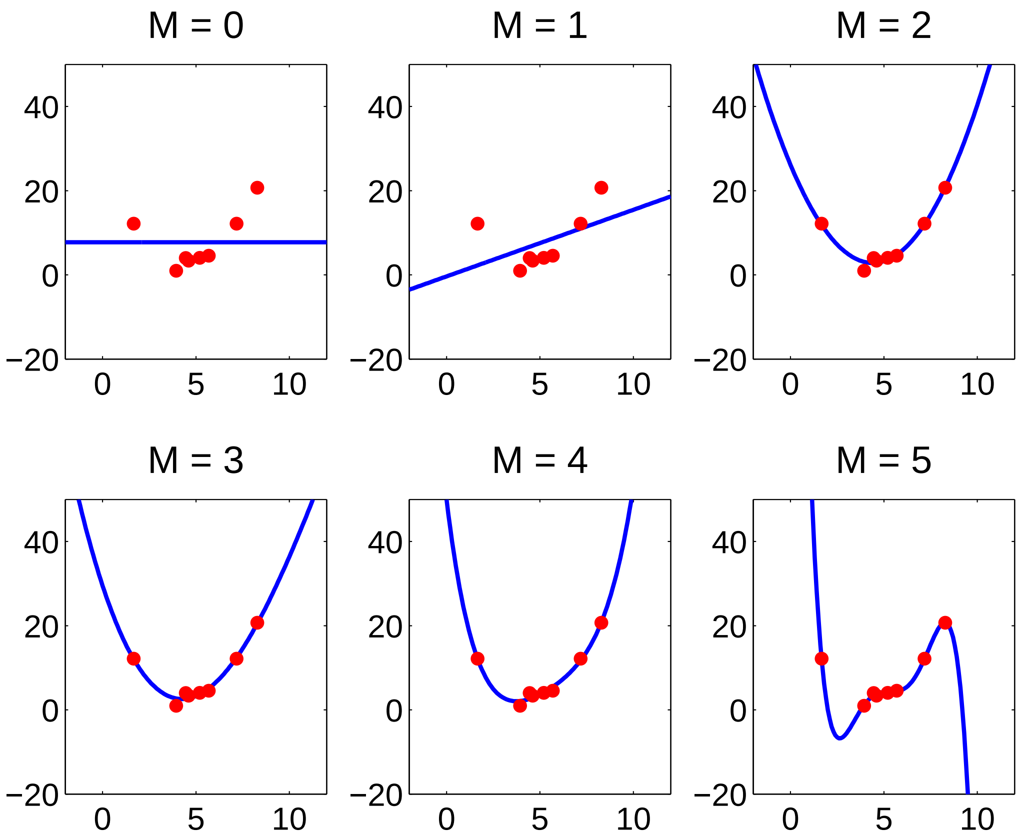
The data set $\mathcal{D}$ consists of N pairs of inputs and corresponding real-valued outputs:

$$\mathcal{D} = \{(\mathbf{x}^{(1)}, y^{(1)}) \ldots, (\mathbf{x}^{(N)}, y^{(N)})\}$$

where $y^{(n)} \in \Re$.

The goal is to predict with accuracy the output given a new input (i.e. to *generalize*).

Linear and Nonlinear Regression



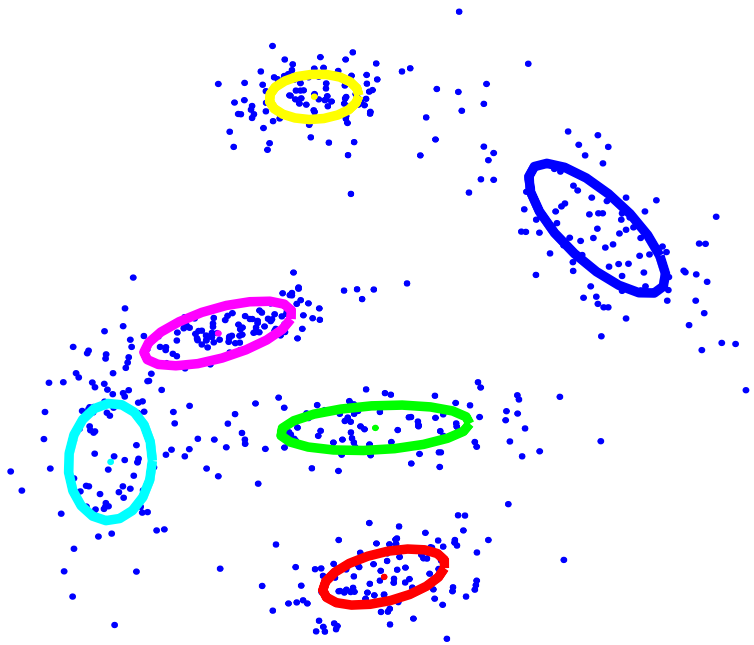
Clustering

Given some data, the goal is to discover "clusters" of points.

Roughly speaking, two points belonging to the same cluster are generally more similar to each other or closer to each other than two points belonging to different clusters.

Examples:

- cluster news stories into topics
- cluster genes by similar function
- cluster movies into categories
- cluster astronomical objects



Basic Rules of Probability

Let X be a random variable taking values x in some set $\mathcal{X}$.

Probabilities are non-negative $P(X = x) \geq 0 \; \forall x$.

Probabilities normalise: $\sum_{x \in \mathcal{X}} P(X = x) = 1$ for distributions if x is a discrete variable and $\int_{-\infty}^{+\infty} p(x)dx = 1$ for probability densities over continuous variables

The joint probability of $X = x$ and $Y = y$ is: $P(X = x, Y = y)$.

The marginal probability of $X = x$ is: $P(X = x) = \sum_y P(X = x, y)$, assuming y is discrete. I will generally write $P(x)$ to mean $P(X = x)$.

The conditional probability of x given y is: $P(x|y) = P(x, y)/P(y)$

Bayes Rule:

$$P(x, y) = P(x)P(y|x) = P(y)P(x|y) \quad \Rightarrow \quad \boxed{P(y|x) = \frac{P(x|y)P(y)}{P(x)}}$$

Warning: I will not be obsessively careful in my use of p and P for probability density and probability distribution. Should be obvious from context.

Information, Probability and Entropy

Information is the reduction of uncertainty. How do we measure uncertainty?

Some axioms (informal):

- if something is certain its uncertainty $= 0$
- uncertainty should be maximum if all choices are equally probable
- uncertainty (information) should add for independent sources

This leads to a discrete random variable X having uncertainty equal to the entropy function:

$$H(X) = -\sum_{x \in \mathcal{X}} P(X = x) \, \log P(X = x)$$

measured in *bits* (**bi**nary digi**ts**) if the base 2 logarithm is used or *nats* (**na**tural digi**ts**) if the natural (base e) logarithm is used.

Some Definitions and Intuitions

- Surprise (for event $X = x$): $-\log P(X = x)$
- Entropy = average surprise: $H(X) = -\sum_{x \in \mathcal{X}} P(X = x)\ \log_2 P(X = x)$
- Conditional entropy

$$H(X|Y) = -\sum_x \sum_y P(x, y) \log_2 P(x|y)$$

- Mutual information

$$I(X;Y) = H(X) - H(X|Y) = H(Y) - H(Y|X) = H(X) + H(Y) - H(X,Y)$$

- Independent random variables: $P(x, y) = P(x)P(y)\ \forall x\, \forall y$

Shannon's Source Coding Theorem

A discrete random variable X, distributed according to $P(X)$ has entropy equal to:

$$H(X) = -\sum_{x \in \mathcal{X}} P(x) \log P(x)$$

Shannon's source coding theorem: Consider a random variable X, with entropy $H(X)$. A sequence of n independent draws from X can be losslessly compressed into a minimum expected code of length $n\mathcal{L}$ bits, where $H(X) \leq \mathcal{L} < H(X) + \frac{1}{n}$.

If each symbol is given a code length $l(x) = -\log_2 Q(x)$ then the expected per-symbol length $\mathcal{L}_Q$ of the code is

$$H(X) + KL(P\|Q) \leq \mathcal{L}_Q < H(X) + KL(P\|Q) + \frac{1}{n},$$

where the relative-entropy or Kullback-Leibler divergence is

$$KL(P\|Q) = \sum_x P(x) \log \frac{P(x)}{Q(x)} \geq 0$$

Take home message: better probabilistic models $\equiv$ more efficient codes

Some distributions

Univariate Gaussian density ($x \in \Re$):

$$p(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

Multivariate Gaussian density ($\mathbf{x} \in \Re^D$):

$$p(\mathbf{x}|\mu, \Sigma) = |2\pi\Sigma|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\mathbf{x}-\mu)^\top \Sigma^{-1} (\mathbf{x}-\mu)\right\}$$

Bernoulli distribution ($x \in \{0, 1\}$):

$$p(x|\theta) = \theta^x (1-\theta)^{1-x}$$

Discrete distribution ($x \in \{1, \dots L\}$):

$$p(x|\theta) = \prod_{\ell=1}^{L} \theta_\ell^{\delta(x,\ell)}$$

where $\delta(a, b) = 1$ iff $a = b$, and $\sum_{\ell=1}^{L} \theta_\ell = 1$ and $\theta_\ell \geq 0 \; \forall \ell$.

Some distributions (cont)

Uniform ($x \in [a, b]$):

$$p(x|a, b) = \begin{cases} \frac{1}{b-a} & \text{if} a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

Gamma ($x \geq 0$):

$$p(x|a, b) = \frac{b^a}{\Gamma(a)} x^{a-1} \exp\{-bx\}$$

Beta ($x \in [0, 1]$):

$$p(x|\alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1 - x)^{\beta-1}$$

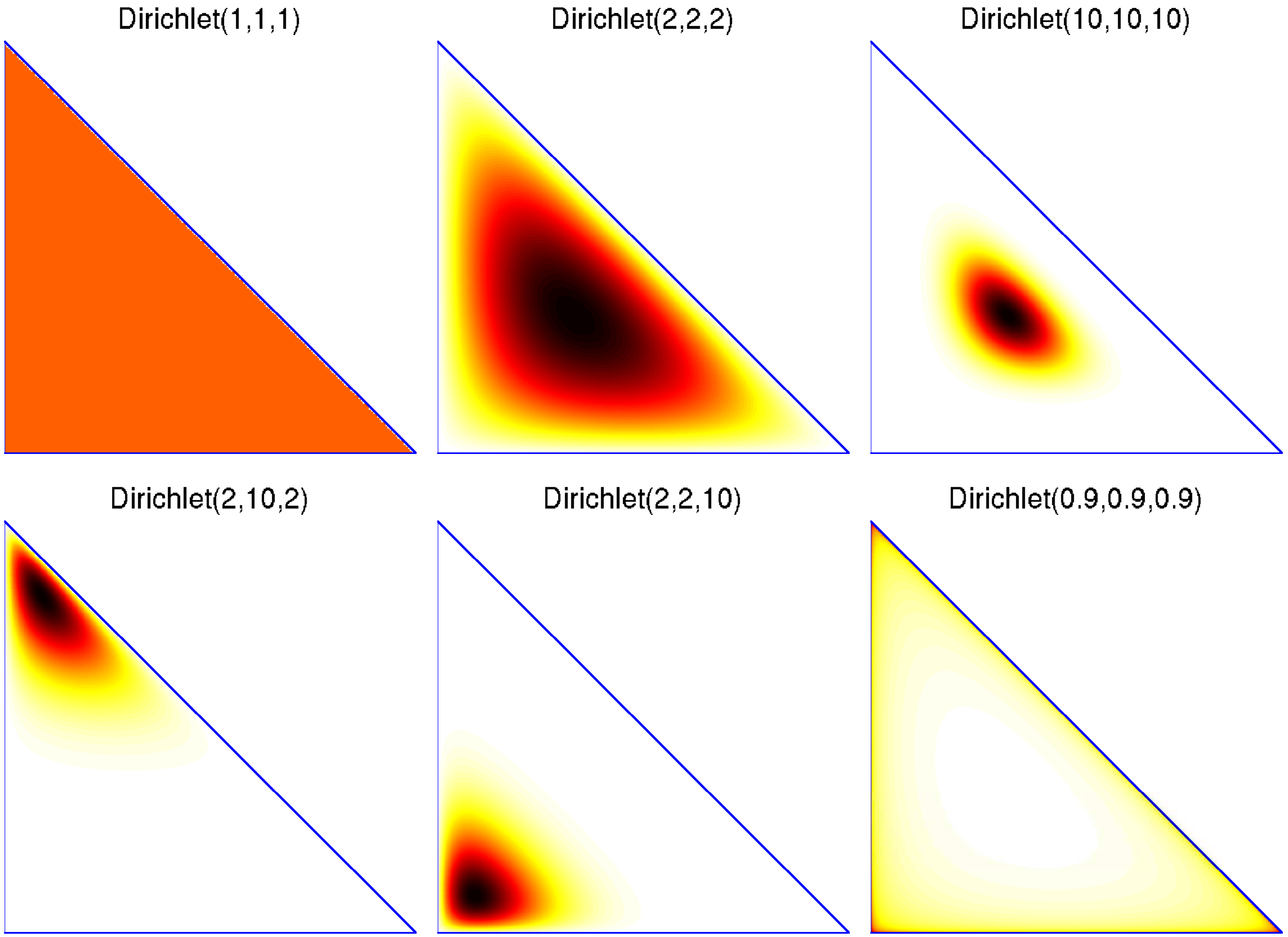
where $\Gamma(z) = \int_0^\infty t^{z-1} e^t dt$ is the gamma function, a generalisation of the factorial: $\Gamma(n) = (n - 1)!$.

Dirichlet ($\mathbf{p} \in \Re^D$, $p_d \geq 0$, $\sum_{d=1}^D p_d = 1$):

$$p(\mathbf{p}|\boldsymbol{\alpha}) = \frac{\Gamma(\sum_{d=1}^D \alpha_d)}{\prod_{d=1}^D \Gamma(\alpha_d)} \prod_{d=1}^D p_d^{\alpha_d - 1}$$

Dirichlet Distributions

Examples of Dirichlet distributions over $\mathbf{p} = (p_1, p_2, p_3)$ which can be plotted in 2D since $p_3 = 1 - p_1 - p_2$:



Other distributions you should know about...

Exponential family of distributions:

$$P(\mathbf{x}|\boldsymbol{\theta}) = f(\mathbf{x})\; g(\boldsymbol{\theta}) \exp\left\{\boldsymbol{\phi}(\boldsymbol{\theta})^\top \mathbf{u}(\mathbf{x})\right\}$$

where $\phi(\boldsymbol{\theta})$ is the vector of *natural parameters*, $\mathbf{u}$ are *sufficient statistics*

- Binomial
- Multinomial
- Poisson
- Student t distribution
- ...

End Notes

It is very important that you *understand* all the material in the following cribsheet:
`http://www.gatsby.ucl.ac.uk/~zoubin/course05/cribsheet.pdf`

Here is a useful statistics / pattern recognition glossary:
`http://research.microsoft.com/~minka/statlearn/glossary/`