

K-L information inequality

With f and g density functions:

$$\begin{aligned}\int \mathbf{log} [f(x)/g(x)] f(x) dx &= -\int \mathbf{log} [g(x)/f(x)] f(x) dx \\ &\geq -\mathbf{log} \left(\int [g(x)/f(x)] f(x) dx \right) \\ &= -\mathbf{log} \left(\int g(x) dx \right) \\ &= -\mathbf{log} (1) \\ &= 0\end{aligned}$$

where the inequality follows is by an application of Jensen's inequality for the concave \mathbf{log} function. Moreover, the inequality is strict unless almost surely w.r.t. the F -distribution, $[g(x)/f(x)] = 1$.