Unsupervised Learning

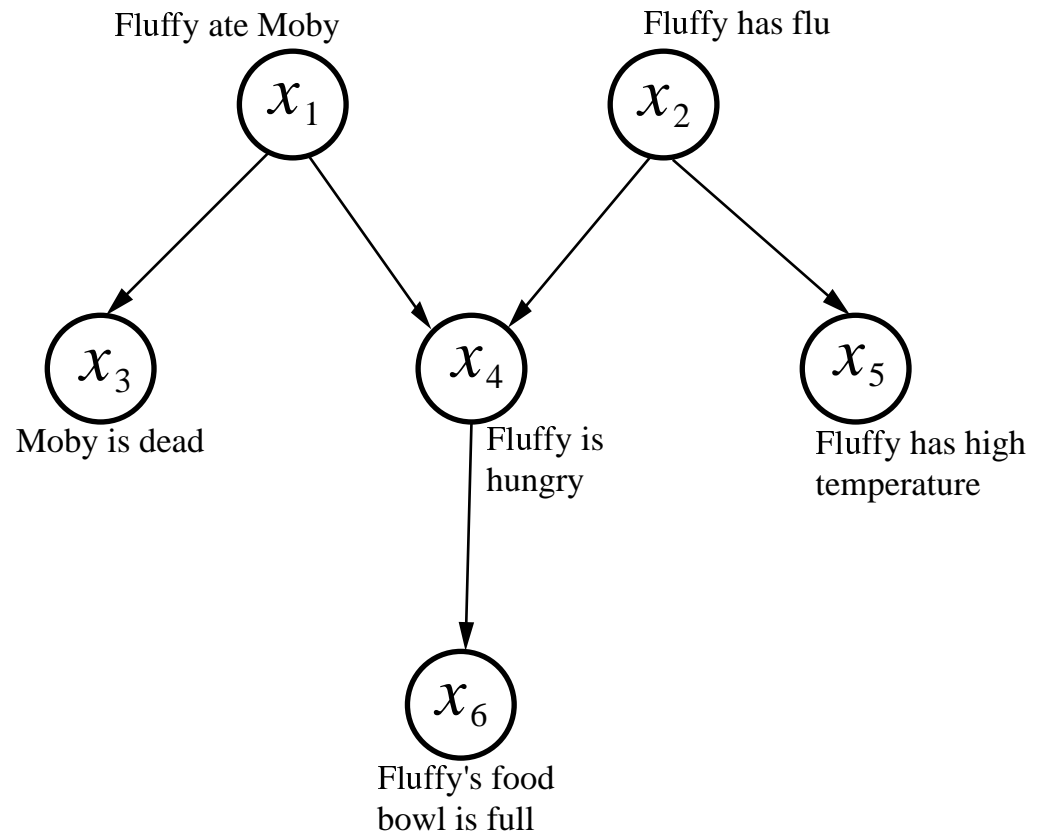## A Belief Propagation Demo

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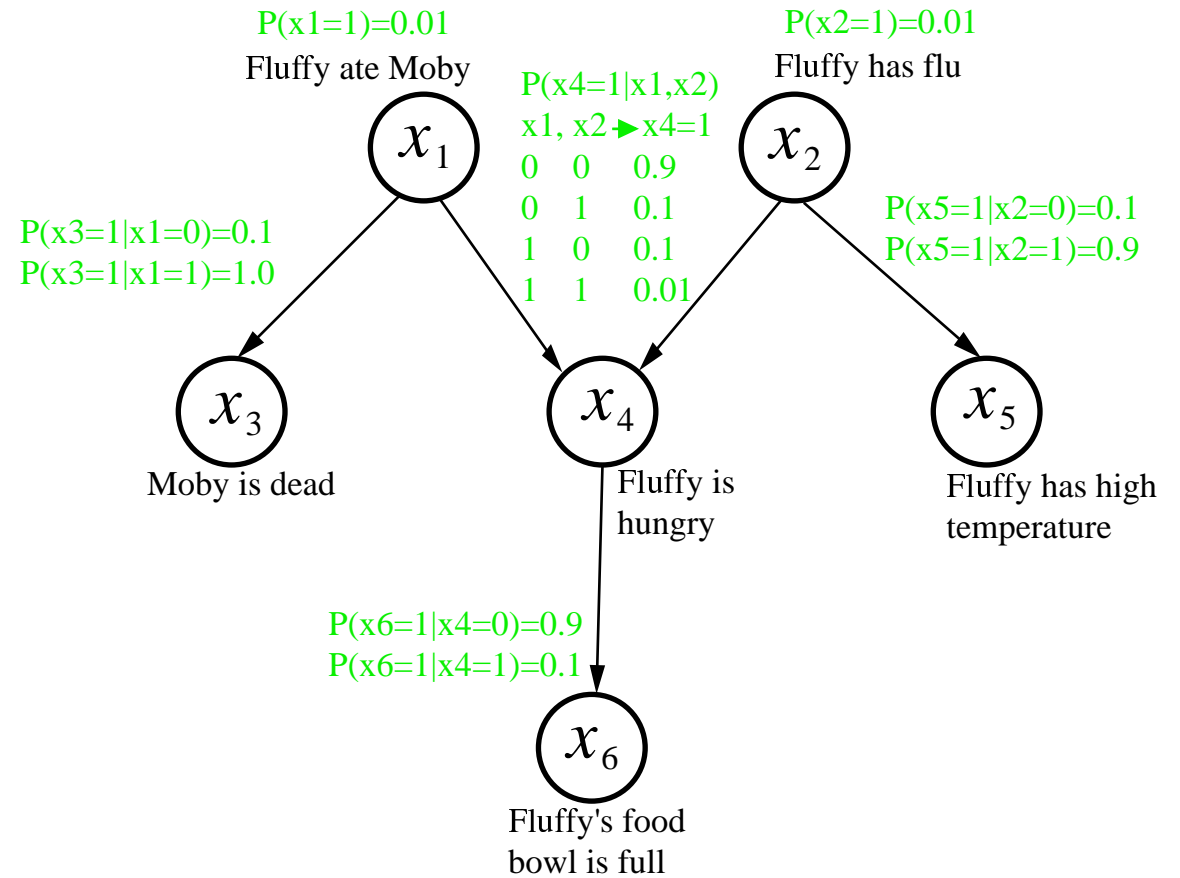
Autumn 2003

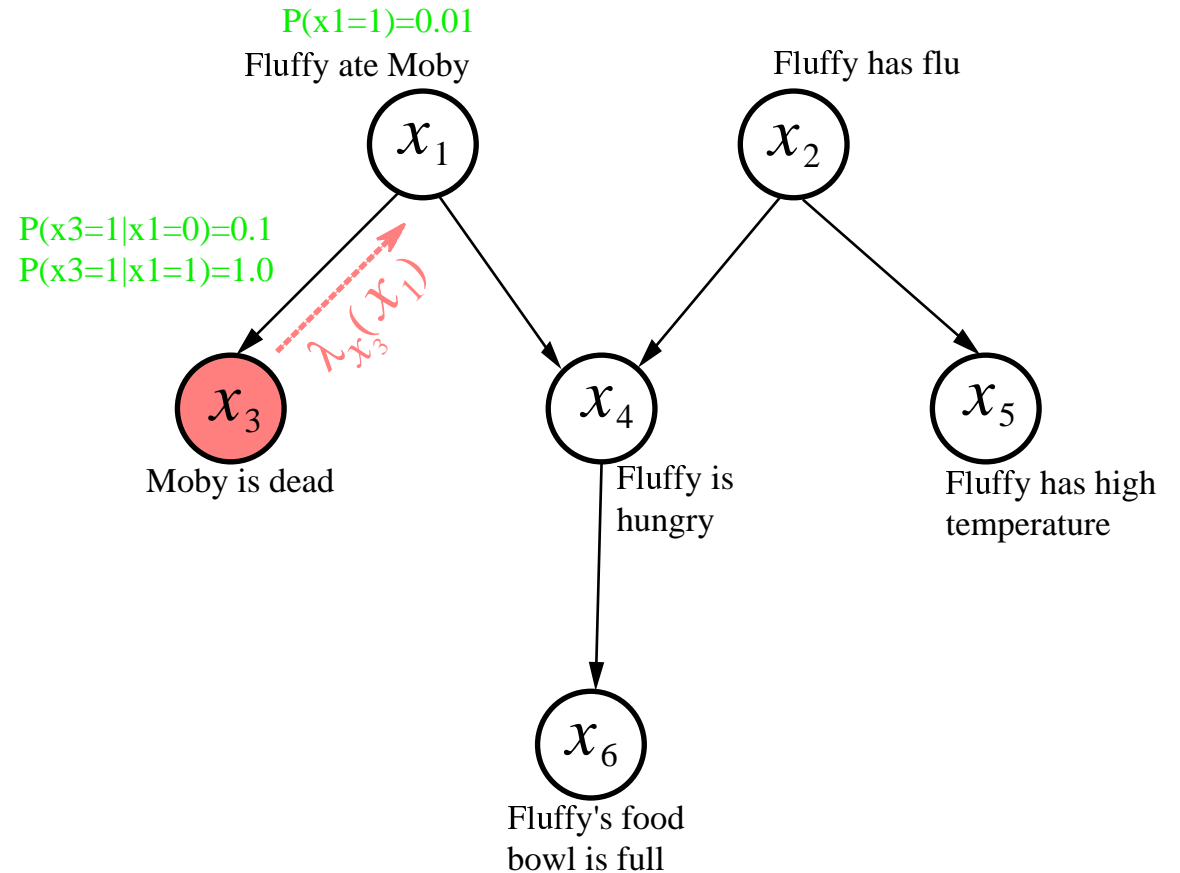# 1. Model Structure

Fluffy = pet cat
Moby = pet fish

Fluffy ate Moby

Fluffy has flu

x_1

x_2

x_3

x_4

x_5

Moby is dead

Fluffy is
hungry

Fluffy has high
temperature

x_6

Fluffy's food
bowl is full

2. Model Parameters

Fluffy = pet cat
Moby = pet fish

P(x1=1)=0.01
Fluffy ate Moby

P(x2=1)=0.01
Fluffy has flu

P(x4=1|x1,x2)

x1,	x2	►x4=1
0	0	0.9
0	1	0.1
1	0	0.1
1	1	0.01

x_1

x_2

P(x3=1|x1=0)=0.1
P(x3=1|x1=1)=1.0

P(x5=1|x2=0)=0.1
P(x5=1|x2=1)=0.9

x_3

x_4

x_5

Moby is dead

Fluffy is
hungry

Fluffy has high
temperature

P(x6=1|x4=0)=0.9
P(x6=1|x4=1)=0.1

x_6

Fluffy's food
bowl is full

3. Propagating Evidence

P(x1=1)=0.01

Fluffy ate Moby

Fluffy has flu

x_1

x_2

P(x3=1|x1=0)=0.1
P(x3=1|x1=1)=1.0

$\lambda_{x_3}(x_1)$

x_3

x_4

x_5

Moby is dead

Fluffy is
hungry

Fluffy has high
temperature

x_6

Fluffy's food
bowl is full
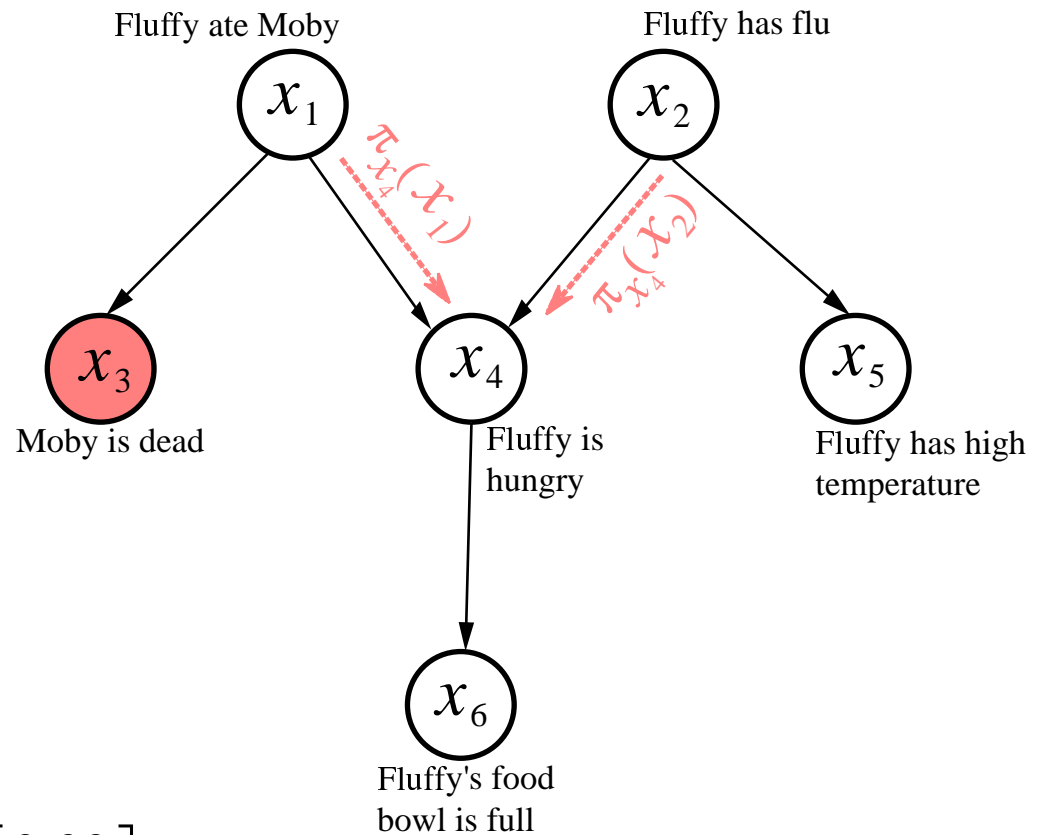
1. Observe "Moby is dead", i.e. $x_3 = 1$

2. Send $\lambda_{x_3}(x_1) \equiv p(e^-_{x_1 \rightarrow x_3}|x_1) = \begin{bmatrix} 0.1 \\ 1.0 \end{bmatrix}$ message $x_3 \rightarrow x_1$

3. $BEL(x_1|x_3 = 1) = \frac{1}{Z} \begin{bmatrix} 0.99 \\ 0.01 \end{bmatrix} \odot \begin{bmatrix} 0.1 \\ 1.0 \end{bmatrix} = \begin{bmatrix} 0.91 \\ 0.09 \end{bmatrix}$
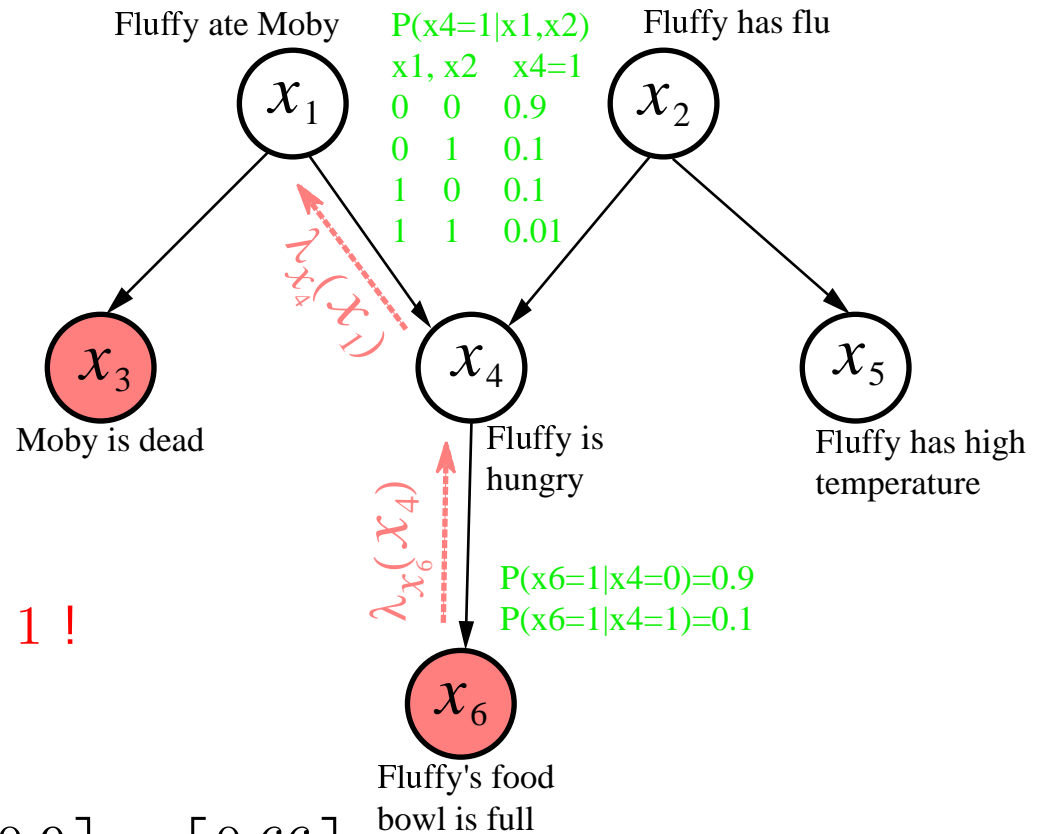
4. Propagating Evidence

Fluffy ate Moby

Fluffy has flu

x_1

$\pi_{x_4}(x_1)$

x_2

$\pi_{x_4}(x_2)$

x_3

Moby is dead

x_4

Fluffy is hungry

x_5

Fluffy has high temperature

x_6

Fluffy's food bowl is full

4. Send $\pi_{x_4}(x_1) \equiv p(x_1|e^+_{x_1 \to x_4}) = \begin{bmatrix} 0.91 \\ 0.09 \end{bmatrix}$

5. Send $\pi_{x_4}(x_2) \equiv p(x_2|e^+_{x_2 \to x_4}) = p(x_2) = \begin{bmatrix} 0.99 \\ 0.01 \end{bmatrix}$ from $x_2 \to x_4$.

6. Compute $\pi(x_4) \equiv p(x_4|e^+_{x_4}) = \sum_{x_1,x_2} p(x_4|x_1,x_2)\pi_{x_4}(x_1)\pi_{x_4}(x_2) = \begin{bmatrix} 0.18 \\ 0.82 \end{bmatrix}$

7. $BEL(x_4|x_3 = 1) = \begin{bmatrix} 0.18 \\ 0.82 \end{bmatrix}$, whereas before observing $x_3 = 1$, $BEL(x_4) = \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix}$.

5. Propagating Evidence

Fluffy ate Moby

x_1

Moby is dead

x_3

P(x4=1|x1,x2)		
x1, x2		x4=1
0	0	0.9
0	1	0.1
1	0	0.1
1	1	0.01

Fluffy has flu

x_2

$\lambda_{x_4}(x_1)$

x_4

Fluffy is hungry

x_5

Fluffy has high temperature

$\lambda_{x_6}(x_4)$

P(x6=1\|x4=0)=0.9
P(x6=1\|x4=1)=0.1

x_6

Fluffy's food bowl is full

8. Observe "Fluffy's Food Bowl is Full" $x_6 = 1$!

9. Send $\lambda_{x_6}(x_4) = \begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix}$ message $x_6 \to x_4$

10. $BEL(x_4 | x_3 = 1, x_6 = 1) = \frac{1}{Z} \begin{bmatrix} 0.18 \\ 0.82 \end{bmatrix} \odot \begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 0.66 \\ 0.34 \end{bmatrix}$

11. Send $\lambda_{x_4}(x_1) = \sum_{x_4} \lambda_{x_6}(x_4) \sum_{x_2} p(x_4 | x_1, x_2) \pi_{x_4}(x_2) = \begin{bmatrix} 0.19 \\ 0.82 \end{bmatrix}$

12. $BEL(x_1 | x_3 = 1, x_6 = 1) = \frac{1}{Z} \begin{bmatrix} 0.99 \\ 0.01 \end{bmatrix} \odot \begin{bmatrix} 0.1 \\ 1.0 \end{bmatrix} \odot \begin{bmatrix} 0.19 \\ 0.82 \end{bmatrix} = \begin{bmatrix} 0.70 \\ 0.30 \end{bmatrix} \Rightarrow$ Fluffy still innocent!