3F3: Signal and Pattern Processing

Lecture 1: Introduction to Machine Learning

Zoubin Ghahramani

zoubin@eng.cam.ac.uk

Department of Engineering University of Cambridge

Lent Term

Machine Learning

Other related terms:

- Pattern Recognition
- Neural Networks
- Deep Learning
- Data Mining
- Data Science
- Statistics
- Artificial Intelligence
- Machine Learning

Learning: The view from different fields

- Engineering: signal processing, system identification, adaptive and optimal control, information theory, robotics, ...
- Computer Science: Artificial Intelligence, computer vision, information retrieval, ...
- Statistics: learning theory, data mining, learning and inference from data, ...
- Cognitive Science and Psychology: perception, movement control, reinforcement learning, mathematical psychology, computational linguistics, ...
- Computational Neuroscience: neuronal networks, neural information processing, ...
- Economics: decision theory, game theory, operational research, ...

Different fields, Convergent ideas

- The same set of ideas and mathematical tools have emerged in many of these fields, albeit with different emphases.
- Machine learning is an interdisciplinary field focusing on both the mathematical foundations and practical applications of systems that learn from data.
- The goal of these lectures: to introduce very basic concepts, models and algorithms.
- Much more on this topic:
 - 4F10: Statistical Pattern Processing
 - 4F13: Machine Learning
 - Advanced Machine Learning (Lent)
 - Information Theory (Lent)
 - Reinforcement Learning and Decision Theory (Lent)

Applications of Machine Learning

Automatic speech recognition



Machine Translation, Dialog Systems, Text modelling and summarisation...

Computer vision: object, face and handwriting recognition, image captioning







"man in black shirt is playing guitar."

"construction worker in orange safety vest is working on road."







"boy is doing backflip on wakeboard."

(NORB image from Yann LeCun, image captioning from Andrej Karpathy and Fei-Fei Li)

Information retrieval and Web Search



Web Pages

Retrieval Categorisation Clustering Relations between pages Personalised search Targeted advertising Spam detection

Financial Prediction and Automated Trading



Medical diagnosis



(image from Kevin Murphy)

Bioinfomatics, Drug discovery, Scientific data analysis



Autonomous Vehicles

Autonomous driving

• ALVINN – Drives 70mph on highways





from 1989 to 2015!

Playing Computer Games



Figure 1: Screen shots from five Atari 2600 Games: (Left-to-right) Pong, Breakout, Space Invaders, Seaquest, Beam Rider

Three Types of Learning

Imagine an organism or machine which experiences a series of sensory inputs:

 $x_1, x_2, x_3, x_4, \ldots$

Supervised learning: The machine is also given desired outputs y_1, y_2, \ldots , and its goal is to learn to produce the correct output given a new input.

Unsupervised learning: The goal of the machine is to build a model of x that can be used for reasoning, decision making, predicting things, communicating etc.

Reinforcement learning: The machine can also produce actions a_1, a_2, \ldots which affect the state of the world, and receives rewards (or punishments) r_1, r_2, \ldots Its goal is to learn to act in a way that maximises rewards in the long term.

Four Problems

Over the next few lectures we will cover these five topics:

- Classification
- Regression
- Clustering
- Dimensionality Reduction

We will make extensive use of probability, statistics, calculus and linear algebra.

Classification

We will represent data by vectors in some vector space. Let x denote a data point with elements $\mathbf{x} = (x_1, x_2, \dots, x_D)$ The elements of x, e.g. x_d , represent measured (observed) features of the data point; D denotes the number of measured features of each point.

The data set \mathcal{D} consists of N pairs of data points and corresponding discrete class labels:

 $\mathcal{D} = \{ (\mathbf{x}^{(1)}, y^{(1)}) \dots, (\mathbf{x}^{(N)}, y^{(N)}) \}$

where $y^{(n)} \in \{1, \ldots, C\}$ and C is the number of classes. The goal is to classify new inputs correctly (i.e. to *generalise*).

Examples:

- spam vs non-spam
- normal vs disease
- 0 vs 1 vs 2 vs 3 ... vs 9

Classification: Example Iris Dataset

3 classes, 4 numeric attributes, 150 instances A data set with 150 points and 3 classes. Each point is a random sample of measurements of flowers from one of three iris species—setosa, versicolor, and virginica—collected by Anderson (1935). Used by Fisher (1936) for linear discrimant function technique.

The measurements are sepal length, sepal width, petal length, and petal width in cm.

Data:

5.1,3.5,1.4,0.2,Iris-setosa
4.9,3.0,1.4,0.2,Iris-setosa
4.7,3.2,1.3,0.2,Iris-setosa
...
7.0,3.2,4.7,1.4,Iris-versicolor
6.4,3.2,4.5,1.5,Iris-versicolor
6.9,3.1,4.9,1.5,Iris-versicolor
...
6.3,3.3,6.0,2.5,Iris-virginica
5.8,2.7,5.1,1.9,Iris-virginica

7.1,3.0,5.9,2.1, Iris-virginica





Regression

Let x denote an input point with elements $\mathbf{x} = (x_1, x_2, \dots, x_D)$. The elements of x, e.g. x_d , represent measured (observed) features of the data point; D denotes the number of measured features of each point.

The data set \mathcal{D} consists of N pairs of inputs and corresponding real-valued outputs:

$$\mathcal{D} = \{(\mathbf{x}^{(1)}, y^{(1)}) \dots, (\mathbf{x}^{(N)}, y^{(N)})\}$$

where $y^{(n)} \in \Re$.

The goal is to predict with accuracy the output given a new input (i.e. to *generalize*).



Linear and Nonlinear Regression

Clustering

Given some data, the goal is to discover "clusters" of points.

Roughly speaking, two points belonging to the same cluster are generally more similar to each other than two points belonging to different clusters.

Examples:

- cluster news stories into topics
- cluster genes by similar function
- cluster movies into categories
- cluster astronomical objects



Dimensionality Reduction

Given some data, the goal is to discover and model the intrinsic dimensions of the data, and/or to project high dimensional data onto a lower number of dimensions that preserve the relevant information.





Basic Rules of Probability

Let X be a random variable taking values x in some set \mathcal{X} . Probabilities are nonnegative, $P(X = x) \ge 0 \forall x$, and normalise: $\sum_{x \in \mathcal{X}} P(X = x) = 1$ for distributions if x is a discrete variable and $\int_{-\infty}^{+\infty} p(x) dx = 1$ for probability densities over continuous variables.

The joint probability of X = x and Y = y is: P(X = x, Y = y).

The marginal probability of X = x is: $P(X = x) = \sum_{y} P(X = x, y)$, assuming y is discrete. I will generally write P(x) to mean P(X = x).

The conditional probability of x given y is: P(x|y) = P(x,y)/P(y)

Sum Rule: $P(x) = \sum_{y} P(x, y)$ **Product Rule:** P(x, y) = P(x)P(y|x) = P(y)P(x|y)

Bayes Rule:

Sum and product rules \Rightarrow

$$P(y|x) = \frac{P(x|y)P(y)}{P(x)} = \frac{P(x|y)P(y)}{\sum_{y'} P(x|y')P(y')}$$

Some distributions

Univariate Gaussian density $(x \in \Re)$:

$$p(x|\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

Multivariate Gaussian density ($\mathbf{x} \in \Re^D$):

$$p(\mathbf{x}|\mu, \Sigma) = |2\pi\Sigma|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\mathbf{x}-\mu)^{\top}\Sigma^{-1}(\mathbf{x}-\mu)\right\}$$

Bernoulli distribution ($x \in \{0, 1\}$):

$$p(x|\theta) = \theta^x (1-\theta)^{1-x}$$

Discrete distribution ($x \in \{1, ..., L\}$):

$$p(x|\theta) = \prod_{\ell=1}^{L} \theta_{\ell}^{\delta(x,\ell)}$$

where $\delta(a, b) = 1$ iff a = b, and $\sum_{\ell=1}^{L} \theta_{\ell} = 1$ and $\theta_{\ell} \ge 0 \ \forall \ell$.

Some distributions (cont)

Uniform $(x \in [a, b])$: $p(x|a,b) = \begin{cases} \frac{1}{b-a} & \text{if } a \le x \le b\\ 0 & \text{otherwise} \end{cases}$ Gamma ($x \ge 0$): $p(x|a,b) = \frac{b^a}{\Gamma(a)} x^{a-1} \exp\{-bx\}$ Beta $(x \in [0, 1])$: $p(x|\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$ where $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$ is the gamma function, a generalisation of the factorial: $\Gamma(n) = (n-1)!.$ Dirichlet ($\mathbf{p} \in \Re^D$, $p_d \ge 0$, $\sum_{d=1}^D p_d = 1$):

$$p(\mathbf{p}|\boldsymbol{\alpha}) = \frac{\Gamma(\sum_{d=1}^{D} \alpha_d)}{\prod_{d=1}^{D} \Gamma(\alpha_d)} \prod_{d=1}^{D} p_d^{\alpha_d - 1}$$

Dirichlet Distributions

Examples of Dirichlet distributions over $\mathbf{p} = (p_1, p_2, p_3)$ which can be plotted in 2D since $p_3 = 1 - p_1 - p_2$:



Other distributions you should know about...

Exponential family of distributions:

$$P(\mathbf{x}|\boldsymbol{\theta}) = f(\mathbf{x}) \ g(\boldsymbol{\theta}) \exp\left\{\boldsymbol{\phi}(\boldsymbol{\theta})^{\top} \mathbf{u}(\mathbf{x})\right\}$$

where $\phi(\theta)$ is the vector of *natural parameters*, **u** are *sufficient statistics*

- Binomial
- Multinomial
- Poisson
- ...

End Notes

It is very important that you *understand* all the material in the following cribsheet: http://mlg.eng.cam.ac.uk/teaching/4f13/cribsheet.pdf

Here is a useful statistics / pattern recognition glossary: http://alumni.media.mit.edu/~tpminka/statlearn/glossary/