4F13 Probabilistic Machine Learning: Coursework #1: Gaussian Processes

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Due: 12:00 noon, Friday Nov 3rd, 2023 online via moodle

Your answers should contain an explanation of what you do, and 2-4 central commands to achieve it (but complete listings are unnecessary and discouraged). You must also give an *interpretation* of what the numerical values and graphs you provide *mean* – why are the results the way they are? Explain your reasoning. **Each question should be labelled and answered separately and distinctly.** Total combined length of answers must not exceed 5 sides of a4 (plus cover page), minimum 11pt font, 1 inch margins.

You need the Gaussian Processes for Machine Learning (GPML) toolbox (version 4.2) for matlab and octave. Get the toolbox and walk through the documentation concerning regression from the Gaussian Process Web site at `www.gaussianprocess.org/gpml/code` Note, that sometimes hyperparameters are encoded using their logarithms (to avoid having to deal with *constrained* optimization for positive parameters), but you will want to report them in their natural domain. All logs are natural (ie, base e). All questions carry approximately equal weight.

a) Load data from `cw1a.mat`. Train a GP with a squared exponential covariance function, `covSEiso`. Start the log hyper-parameters at `hyp.cov = [-1 0]; hyp.lik = 0;` and minimize the negative log marginal likelihood. Show the 95% predictive error bars. Explain the values of the optimized hyperparameters and the shape of the predictive error bars.

b) How can you find out whether the hyper parameter optimum is unique, or whether there may be other local optima? If there are local optima, find some, and explain what the model is doing in each case. Which fit is best, and why? How confident are you about this and why?

c) Train instead a GP with a periodic covariance function. Compare the behaviour of the error-bars with a). Do you think the data generating mechanism (apart from the noise) was really strictly periodic[1] (in the interval where data was observed)? Quantify how confident you are about this, and why?

d) Generate random (essentially) noise free functions evaluated at `x = linspace(-5,5,200)';` from a GP with the following covariance function: `{@covProd, {@covPeriodic, @covSEiso}}`, with covariance log hyperparameters `hyp.cov = [-0.5 0 0 2 0]`. In order to apply the Cholesky decomposition to the covariance matrix, you may have to add a small diagonal matrix, for example `1e-6*eye(200)`, why? Plot some sample functions. Carefully explain the relationship between the properties of those random functions and the form of the covariance function.

e) Load `cw1e.mat`. This data has 2-D input and scalar output. Visualise the data, for example using `mesh(reshape(x(:,1),11,11),reshape(x(:,2),11,11),reshape(y,11,11));` Rotate the data, to get a good feel for it. Compare two GP models of the data, one with `covSEard` covariance and the other with `{@covSum, {@covSEard, @covSEard}}` covariance. For the second model be sure to break symmetry with the initial hyperparameters (eg by using `hyp.cov = 0.1*randn(6,1);`).

   Compare the models: give a careful quantitative interpretation of the relationship between data fit, model complexity and marginal likelihood for each of the two models and compare.

[1] By strictly periodic, is meant a function where the exists a p such that $f(x) = f(x+p)$ for all x, not just a a function which goes up and down.