

# Finite and infinite basis GPs

Hong Ge

Oct, 2025

# Key concepts

- Should we use finite or infinite models?
- GPs are a fancy way of using infinite models, but
  - will it actually make any difference in practise?
- finite models correspond to much stronger assumptions about the data
- therefore, we don't want to use finite models
- a GP with squared exponential covariance function corresponds to an infinite linear in the parameters model with Gaussian bumps **everywhere**
- illustrate the difference

## Cromwell's dictum

*I beseech you, in the bowels of Christ, consider it possible that you are mistaken*

— Oliver Cromwell, 1650

# From infinite linear models to Gaussian processes

Consider the class of functions (sums of squared exponentials):

$$\begin{aligned}f(x) &= \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=-N/2}^{N/2} \gamma_n \exp(-(x - \frac{n}{\sqrt{N}})^2), \text{ where } \gamma_n \sim \mathcal{N}(0, 1), \forall n \\&= \int_{-\infty}^{\infty} \gamma(u) \exp(-(x - u)^2) du, \text{ where } \gamma(u) \sim \mathcal{N}(0, 1), \forall u.\end{aligned}$$

The mean function is:

$$\mu(x) = \mathbb{E}[f(x)] = \int_{-\infty}^{\infty} \exp(-(x - u)^2) \int_{-\infty}^{\infty} \gamma(u) p(\gamma(u)) d\gamma(u) du = 0,$$

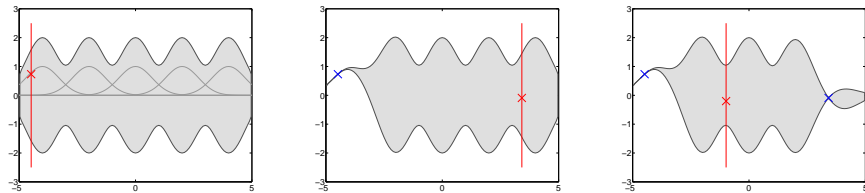
and the covariance function:

$$\begin{aligned}\mathbb{E}[f(x)f(x')] &= \int \exp(-(x - u)^2 - (x' - u)^2) du \\&= \int \exp(-2(u - \frac{x + x'}{2})^2 + \frac{(x + x')^2}{2} - x^2 - x'^2) du \propto \exp(-\frac{(x - x')^2}{2}).\end{aligned}$$

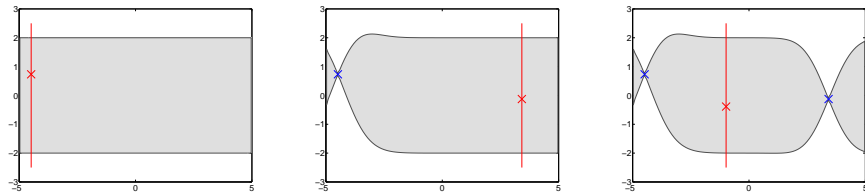
Thus, the squared exponential covariance function is equivalent to regression using infinitely many Gaussian shaped basis functions placed everywhere, **not just at your training points!**

# Using finitely many basis functions may be dangerous!(1)

Finite linear model with 5 localized basis functions)

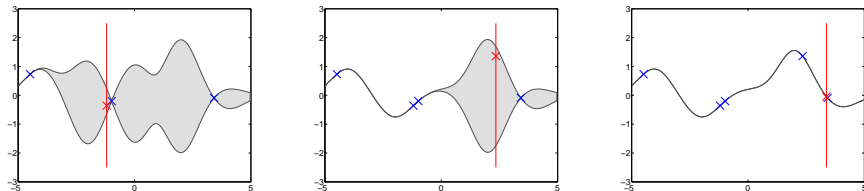


Gaussian process with infinitely many localized basis functions

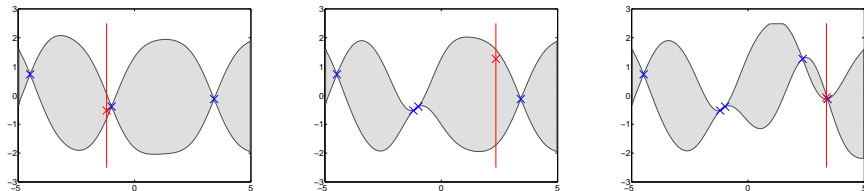


# Using finitely many basis functions may be dangerous!(2)

Finite linear model with 5 localized basis functions)

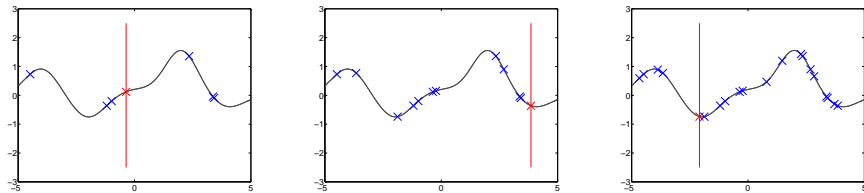


Gaussian process with infinitely many localized basis functions



# Using finitely many basis functions may be dangerous!(3)

Finite linear model with 5 localized basis functions)



Gaussian process with infinitely many localized basis functions

